Research on Pressure Stability Control of High-pressure Common Rail System Based on Differential Equation Optimization Model

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Abstract. To improve the accuracy of fuel injection in high-pressure common rail systems of diesel engine, a mathematical model based on differential equations was established in view of the process of fuel entering and ejecting the common rail pipe, and the pressure variation law under working condition in the common rail tube was simulated in this study. Aiming at reducing the fuel pressure fluctuations in the system, the optimal angular velocity of the cam in high-pressure pump is solved by binary search algorithm, and the optimal control strategies for maintaining the dynamic and stability of pressure are given as well. Numerical simulation results show that with the presented strategies precise control of the pressure can be achieved in the common rail system, helping to improve the efficiency of the diesel engines.

1. Introduction

As the core of the diesel engines, high-pressure common rail system has the advantages of high injection pressure and flexible and adjustable quantity of fuel injection, which significantly improve the economy, power and emission performance of diesel engines, and now it has become the world's most recognized diesel injection system [1-3]. The intermittent process of fuel entering and ejecting the common rail pipe will cause pressure fluctuations in the common rail pipe, which can affect the accuracy of the fuel injection quantity from the spray hole. Therefore, the system pressure control includes the rail pressure stability control and the rail pressure responsiveness control, which are two important parts while formulating common rail system control strategies: The purpose of the rail pressure stability control is to reduce the rail pressure oscillations during system operation, furthermore, decrease the deviation of the injected fuel quantity from the target value; And the rail pressure responsiveness control aims to realize the real-time adjustment of the rail pressure, that is, the present rail pressure is supposed to be changed to the target value quickly and accurately when the engine needs to switch working conditions, the purpose of which is to meet the fuel supply requirements of the engine at different speeds [4-6].

The study on the rail pressure control strategy of the common rail system has received increasing attention from researchers, who have mostly focused on software simulation and establishment of mathematical models. Aboelfadl et al.[7] represented a one-dimensional simulation modelling of the high-pressure common rail system in MATLAB/Simulink software environment, realizing the rough modeling and optimization of rail pressure control and application layer; And in [8], a three-dimensional computational fluid dynamics simulation of water hammer was performed to obtain a deep insight into the transient three-dimensional effects, aiming at better understand the transient effect of water hammer

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pressure triggered by injection in high-pressure common rail system for precise control of injection. However, these simulation models of the system had been oversimplified and ignored the dynamic relationship between the different structures. In [9,10], three mathematical sub-models of common rail system were formulated: hydraulic sub-model, mechanical sub-model and electromagnetic sub-model, and the effect of pressure bottoming on the quality of injected fuel was studied. Most of these models, however, are only suitable for dynamic simulation of the pressure in system, which are lacking in the optimization of specific pressure control strategies. Hence, how to simulate the state of the common rail systems through mathematical models and provide reliable rail pressure control strategies are still problems worth discussing.

The pressure control of the common rail system is one of the most important aspects of the system control, and it is also the basis for realizing the technical innovation of the diesel engine. The current studies on the common rail system are mostly limited to the modeling and simulation analysis of sub-modules. Based on these, this paper will analyze and derive mathematical models on pressure control of the common rail system, which are concise and efficient, to simulate the working process of each component in the system and the pressure change law as well, then finally get the optimization strategies on stabilizing and regulating pressure in the common rail.

2. System working principle

The structure of a common high-pressure common rail system, which is mainly composed of high - pressure pump (HPP), common rail pipe and injector, is shown in Figure 1. In the actual working process, the pressurized fuel is constantly supplied into the common rail pipe by the HPP. For the injector has an extremely high requirement on pressure stability of the fuel supply, the HPP is designed to be connected to the injector through a common rail pipe, which can reduce the pressure fluctuation caused by the discontinuous fuel supply during system operation [11].

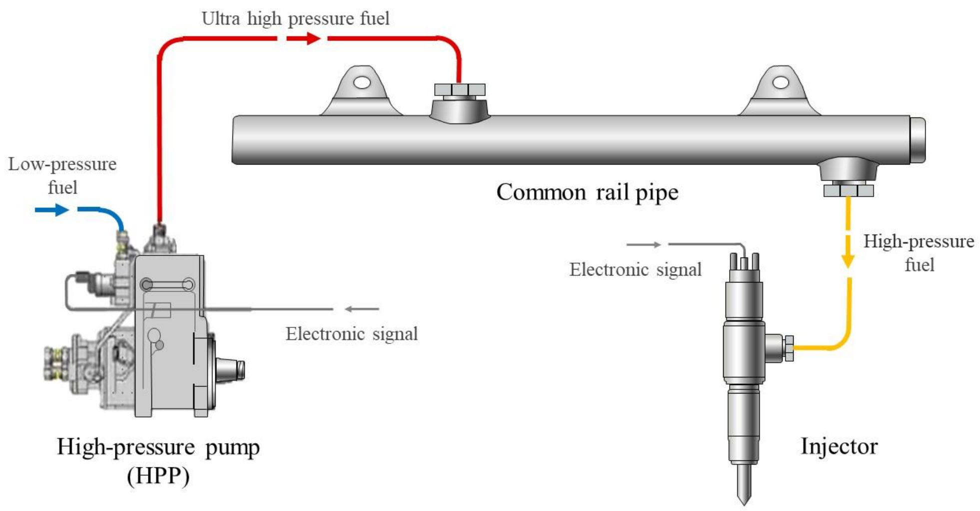

Figure 1. Structure of high-pressure common rail system.

The cam in the HPP is driven by the engine to keep the plunger reciprocate up and down in the cavity: when the plunger moves from lower dead center to top dead center under the driven of the cam, the fuel pressure in the plunger cavity can be rapidly increased. When the pressure in the cavity is higher than it in the common rail pipe, the HPP outlet valve will be opened, then high-pressure fuel can be sent into the common rail pipe. When the plunger continues to descend from top dead center, a low pressure will be formed in plunger cavity, and then the low-pressure fuel will be accumulated in the plunger cavity of the HPP, thus the fuel is periodically and cyclically supplied. The structure of the HPP is shown in Figure 2.

The injector works periodically under the control of the electronic control unit (ECU), and it is responsible for injecting the fuel in the common rail pipe into the cylinder of the engine. When the ECU does not send out a control signal, the solenoid valve in the injector will not operate and the fuel injection port will keep closed. When the injector receives an electric signal from the ECU, the solenoid valve is rapidly raised by the power drive circuit, and then start to inject fuel. The structure of the electronic controlled injector is shown in Figure 3. The fuel injection timing of the injector is determined by the start time of the high-speed solenoid valve, while the fuel injection quantity from the injector is determined by the conduction time of the high-speed solenoid valve [12].

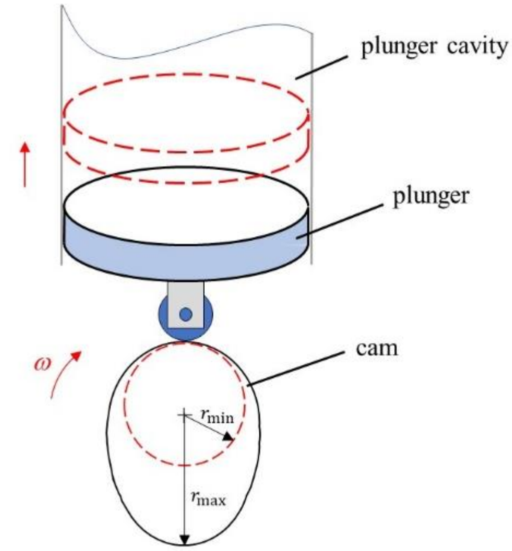

Figure 2. Structure of high-pressure pump.

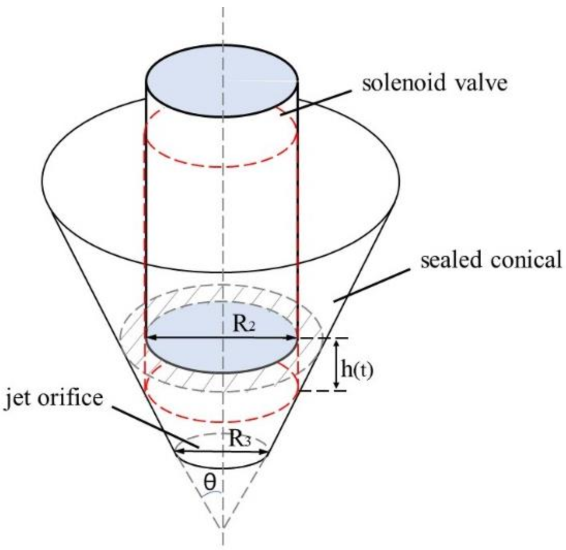

Figure 3. Structure of injector.

3. Modeling ideas and assumptions

According to the actual working principle of the common rail system, the fuel injection process of the injector should meet the fuel consumption demand of the engine in real time. When working situation of the engine is determined, the pressure in the common rail pipe only depends on the oil supply of the HPP, which is controlled by the speed of cam in the HPP. Therefore, in order to obtain the best pressure stabilization control strategy of the high-pressure common rail system, it is quite necessary to establish an optimization model with the cam speed ω as the decision variable, and use differential equations to simulate the transmission process of fuel in each structure of the system. At the same time, setting out to seek the constraints of the optimization model by analyzing the working principles of the three important components: the HPP, the common rail pipe and the injector.

In order to facilitate the indaped study and make it convenient to simulate the working condition of the system, the following assumptions are made: 1) The friction between the fuel flow and the various channels is negligible; 2) The cam and the container wall where the fuel is located are rigid structures, deformation can be ignored; 3) In the same volume, the density and pressure of the fuel are equal everywhere; 4) Ignore the temperature change caused by the change in fuel pressure.

4. Rail pressure stability control model

4.1. Objective function

Based on the analysis of the working principle and performance indicators of the high-pressure common rail system, the control objective of the pressure stability control model is to reduce the pressure fluctuations in the common rail tube. It is assumed that P_{aim} is the desired pressure stability control target value while $P_2(t)$ is the internal pressure of the common rail pipe at time t, T is a complete control cycle. Then the objective function is

$$\min\int_0^T \left(P_2(t)-P_{aim}\right)^2 dt \tag{1}$$

4.2. Constraint conditions

4.2.1. High-pressure pump control. During the operation of the HPP, the plunger cavity can be regarded as a container whose volume changes with time. According to the mass conservation principle of fuel in the cavity, the following equation is obtained

$$\frac{dP_1}{dt}=\frac{E}{V_1}\left(\frac{dV_1}{dt}+Q_{in1}-Q_{out1}\right). \tag{2}$$

In which, P_1 is the pressure inside the plunger cavity, V_1 is the volume of the plunger cavity, Q_{in1} and Q_{in2} denote the flows of fuel flowing into and out of the plunger cavity, and E is the elastic modulus of the fuel. In this study, the relationship between the plunger cavity volume V_1 and rotation angle of the cam ω can be expressed as

$$V_1=\pi\left(\frac{R_1}{4}\right)^2\left[H-h(\varphi)\right]=\pi\left(\frac{R_1}{4}\right)^2\left[H-h(\omega t)\right]. \tag{3}$$

Where $h(\omega t)$ is the rising distance of the plunger relative to the lower dead center at time t, and R_1 is the inner diameter of the plunger cavity.

On the basis of the integral relationship between fuel flow and volume and Bernoulli Equation, the equations of fuel flow into and out of the plunger cavity can be respectively calculated as follows:

$$V_{in1} = \int_{t_0}^{t_0+T_1} Q_{in1} dt \,, \tag{4}$$

$$Q_{out1} = \varepsilon_\tau \mu S_1 \sqrt{\frac{2\left|P_1 - P_{p2}\right|}{\rho_1}} \,. \tag{5}$$

Where V_{in1} denotes the volume of fuel entering the pump within a cycle T_1 of the cam movement, μ is the flow coefficient, S_1 is the cross-sectional area of the oil outlet of the HPP, P_1-P_{p2} is the pressure difference across the oil outlet, and ρ_1 is the density of high-pressure fuel in the plunger cavity, ε_τ is a step function: ε_τ =1 while P_1>P_{p2}, otherwise ε_τ =1.

4.2.2. Pressure control in common rail. According to the mass conservation of fuel in the common rail pipe during time period Δt, we get

$$m(t+\Delta t) - m(t) = \rho_1 Q_{in2} \Delta t - \rho_2 Q_{out2} \Delta t \,. \tag{6}$$

From the relationship between fuel quality and density, we can obtain that the change in fuel density during time Δt is

$$\rho_2(t+\Delta t) - \rho_2(t) = \frac{1}{V}\left[m(t+\Delta t) - m(t)\right]. \tag{7}$$

Plugging (7) into the equation for (8), and in an extremely short period of time ($\Delta t \to 0$), these equations can be transformed into a partial differential equation of fuel density in the common rail tube with respect to time:

$$\frac{\partial \rho_2(t)}{\partial t} = \frac{1}{V}\left[\rho_1 Q_{in2} - \rho_2 Q_{out2}\right]. \tag{8}$$

According to literature [6], the amount of change in pressure is proportional to the amount of change in density, that is

$$\frac{dP(t)}{d\rho(t)} = \frac{E}{\rho(t)} \,. \tag{9}$$

Integrating (8) and (9), after rearranging we obtain partial differential equation of fuel density with respect to time in common rail pipe:

$$\frac{\partial P_2}{\partial t} = \frac{E}{V_2}\left(Q_{in2} - Q_{out2}\right). \tag{10}$$

The volume flow Q_{out1} of the oil discharged by the HPP is equal to the volume flow Q_{in2} of the fuel pumped into the common rail pipe, that is Q_{out1} = Q_{in2}. Therefore, we get the following differential equation which can finally simulate the pressure fluctuation in common rail pipe:

$$\frac{\partial P_2}{\partial t} = \frac{E}{V_2}\left(Q_{out1} - Q_{out2}\right). \tag{11}$$

Because the injection pressure of the injector is determined by the pressure in the common rail pipe, to ensure the accuracy of the fuel injection pressure, the pressure fluctuation range in the common rail pipe must be limited to an allowable range. Assume that during the stable operation of the high-pressure common rail system, the maximum and minimum pressure fluctuations in the common rail pipe are $P(t)_{max}$ and $P(t)_{min}$ respectively. Formulating a constraint on the magnitude of pressure fluctuations:

$$\min\left\{\left|P(t)_{\max} - P_{aim}\right|, \left|P(t)_{\min} - P_{aim}\right|\right\} \le eps\,. \tag{12}$$

4.2.3. Fuel injector control. According to the analysis of common injector nozzle structure in Figure 2, it can be confirmed that the actual effective area of fuel injection may have two values, namely the cross-sectional area of the gap between the solenoid valve and the sealed conical, or area of the small hole at the bottom of the injector, whose specific value is determined by the movement state of the solenoid valve. On the basis of the geometric relation, the cross-sectional area $A_1(t)$ of the gap between the solenoid valve and the sealed conical can be calculated by

$$A_1(t) = \pi\left[\left(\frac{R_2}{2} + h(t)\tan\theta\right)^2 - \left(\frac{R_2}{2}\right)^2\right] \tag{13}$$

where R_2 is the outer diameter of the solenoid valve, and $h(t)$ is the distance that the solenoid valve rises after receiving the signal from the ECU. Area of the small hole at the bottom of the injector is calculated by $A_2 = \pi\left(\frac{R_3}{2}\right)^2$,then the actual injection area can be determined by $A(t) = \min\,\{A_1(t), A_2\}$.

Assume that the pressure in the common rail pipe is $P_2(t)$, and the pressure in the cylinder connected to the injector is P_{cy}. The flow of the common rail pipe is determined by the position of the solenoid valve of the injector as well as the difference between the internal and external pressure of the system. According to Bernoulli Equation, the fuel injection flow of the injector is calculated by

$$Q_{out2} = \varepsilon_\tau \mu A(t)\sqrt{\frac{2\left|P_2(t) - P_{cy}\right|}{\rho_2}} \tag{14}$$

where ε_τ is a step function: ε_τ =1 while $P_1 > P_{p2}$, otherwise ε_τ =1.

4.3. Pressure regulation model

The diesel engines have distinct requirements in different working stages for oil volume and oil pressure, so the pressure in the common rail pipe needs to be adjusted in real time by changing the speed of the cam in the HPP. Assume that the pressure P_0 in the pipe need to be increased to the target value P_{aim} within the setting time. The amount of oil fed into the common rail pipe exceeds the amount of oil ejected in this process. Let V_2 be the volume of the common rail pipe. According to equation (9), density of fuel increases as density pressure increases, and density increment can be calculated by:

$$\Delta\rho_2 = \frac{1}{V_2}\left(\int_0^t \rho_1(t)V_{in2}(t)dt - \int_0^t \rho_3(t)V_{out2}(t)dt\right). \tag{15}$$

Then let the fuel density in the tube initially be ρ_0, and integrate equations (9) and (15), get the pressure regulation model in common rail pipe:

$$\mathrm{P}_{aim} = \frac{E}{\rho_2(t)}\left[\rho_0 + \frac{1}{V_2}\left(\int_0^t \rho_1(t)V_{in2}(t)dt - \int_0^t \rho_3(t)V_{out2}(t)dt\right)\right]. \tag{16}$$

5. Numerical simulation and model solving

The data used in numerical simulation are derived from the problem A of China Undergraduate Mathematical Contest in Modeling 2019, which provided the data of a real high-pressure common rail system, including the physical parameters of the system components, the cam edge curve data of the HPP, movement curve data of the solenoid valve in injector, and the data about the relationship between the elastic modulus and pressure. And the pressure in the common rail pipe is supposed to be controlled stabilized at 100 MPa.

5.1. Data fitting

5.1.1. Fuel pressure and elastic modulus. Carrying on quadratic fitting on the data of the relationship between the elastic modulus E and pressure P of the fuel given in the contest, and we can get the following quadratic function:

$$E = 0.008P^2 + 1.2P + 1900. \tag{17}$$

Integrating equation (9) produces

$$\int \frac{dP}{E} = \int \frac{d\rho}{\rho} + c. \tag{18}$$

By substitution of the initial value $P_0=100$MPa, $\rho_0=0.85mg/mm^3$ in the common rail pipe into the equation (16), we obtain (19), we get the algebraic relationship between fuel pressure P and density ρ:

$$\rho = -6.54\times10^{-7}P^2 + 0.00062P + 0.8. \tag{19}$$

5.1.2. Cam profile curve. Fitting the curve of the cam profile we have the relationship between the pole diameter r and the rotation angle φ of the cam

$$r = 2.413(2+\cos\varphi). \tag{20}$$

Then a rectangular coordinate system is established with a longitudinal axis parallel to the direction of movement of the plunger, a horizontal axis perpendicular to the direction of movement of the plunger, and the origin of the cam rotation axis. According to the conversion relationship between polar coordinates and rectangular coordinates:

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}, \tag{21}$$

when the cam rotation angle is φ_k, the ordinate corresponding to each point on the cam contour line in the rectangular coordinate system is $Y_k=[y_{1,k},y_{2,k},\ldots,y_{n,k}]$, If the base circle radius of cam is r_0, then when the cam angle is φ_k, the lift distance of the plunger can be calculated from

$$h(\varphi_k) = \max(Y_k) - r_0\ , \tag{22}$$

5.2. Binary search algorithm
Due to the complexity of the mathematical model, the optimal solution can hardly be obtained by the method of calculating the extreme value of the derivative, so a binary search algorithm is adopted by MATLAB programming. The specific steps of the algorithm are as follows:

a) Set the initial step size of cam angular velocity to 0.01, and the angular velocity search range from 0.001π to π, with the unit in *rad/ms*;

b) Use the cam angular velocity at previous moment to calculate the pressure in the common rail of the system at the current angular velocity through the Fourth-Order Runge-Kutta;

c) Set the working time of the system to 100s, then compute the mathematical expectation E of the pressure in the common rail during the last 1s. If $E > P_{aim}$, the search range will be reduced to the left half interval, otherwise the search range will be the right half interval;

d) Determine whether the calculation time has reached the predetermined value. If not, increase the step size and return to step b). If it does, the algorithm will be terminated.

6. Results

6.1. Pressure stabilization model
The solving results of the pressure stabilization model showed that when the angular velocity of the cam was around 0.0274*rad/ms*, the stability of the rail pressure under the given stable working condition could be guaranteed while the pressure waved around the target control value, i.e.100 MPa, and the pressure fluctuation range was within the allowable range (steady-state error was less than 4 MPa). During the 1000s control period, the pressure fluctuation in the tube is shown in Figure 4.

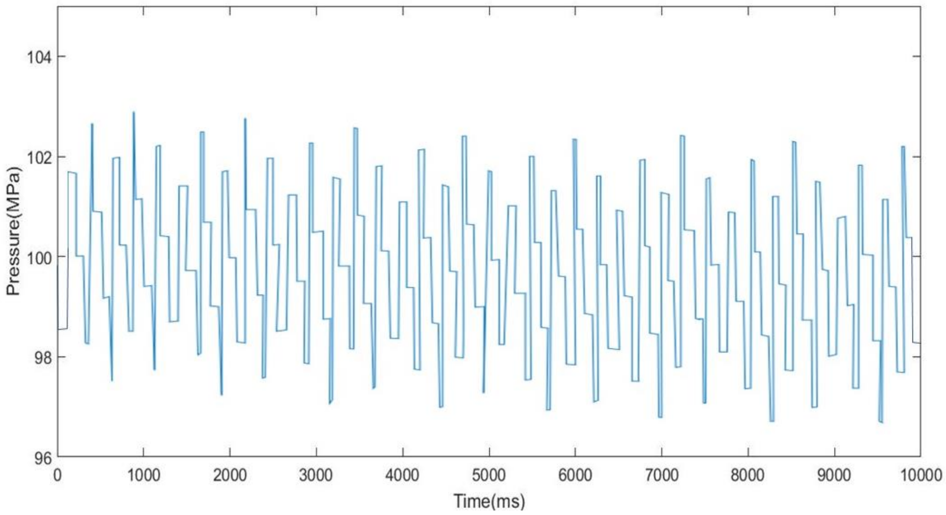

Figure 4. Pressure fluctuation in common rail pipe during control period.

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As can be seen from the Figure 4, the pressure in the common rail pipe only has an extremely slight downward trend throughout the whole solution period, and the amplitude of the up and down fluctuations is within 4 MPa, which accurately meets the pressure control requirements of the high - pressure common rail system. Therefore, the solution result is reliable, showing the model we built is strong and effective.

6.2. Pressure regulation model

According to the solution of the pressure regulation model, if the internal pressure of the common rail pipe need to be increased from 100 MPa to 150 MPa in 5 seconds, the cam speed during the pressure adjustment process have to be increased from the original 0.0274*rad/ms* to 0.0563*rad/ms*, and the pressure in the tube at the 5th second is 149.96 MPa. If the above purpose is supposed to be achieved in 10 seconds, the angular velocity of the cam needs to be adjusted to 0.0463*rad/ms*, and the pressure in the tube at the 10th second is 150.05 MPa. The pressure change situation in the common rail pipe during the setting control time is presented in Figure 5.

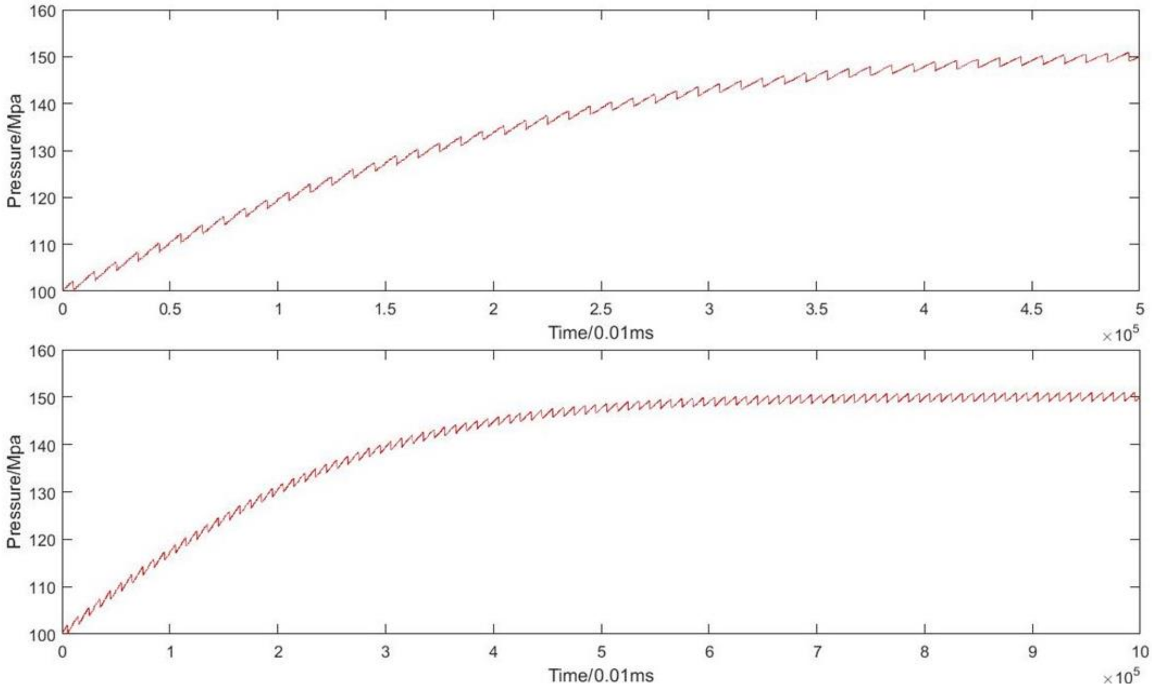

Figure 5. Pressure change in common rail pipe during regulation period.

7. Conclusions

Realizing the precise pressure control of the common rail systems is the key to improving the performance of diesel engines, which directly affects the starting, idling and acceleration properties of diesel engines. Based on the working rules of the common rail system, this paper analyzed and established a pressure stabilization model and a pressure regulation model of the high-pressure common rail system based on the laws of conservation of mass, Newton's law and Bernoulli Equation. Numerical simulation showed that the rail control strategies obtained through this mathematical model can effectively regulate the pressure in the common rail pipe, meeting the actual needs of the project, ensuring the working performance of the diesel engine, and providing a certain theoretical basis for the research of the rail pressure control strategies. And the strategies proposed in this paper also provided a reference for the structural parameter optimization design of common rail pipe and injector for their generality.

Based on the development status of diesel engines, the models presented in this paper are suitable for the most common high-pressure common rail systems. For the common rail system with multiple injectors, and even the recently proposed multi-hole nozzle system, the models need to be improved. In addition, the pressure changes caused by vortexes because of fuel flow in the system will be the direction of further research.

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